Surfaces: Graphs, Groups & applications 4/3/2017

Planned Topics:

1) Classification of surfaces (John Stillwell / classical topology & combinatorial gr. theory, John Lee / Intro to Topological manifolds)
2) The arc comlex (Harer) (Hatcher 1991)
3) Surfaces & commutator length (Culler 1981)
4) Random Gaussian Matrices & enumeration of Graphs on Surfaces. (Lando-Zvonkin / graphs on surfaces & Applications (chapter 3))
5) Mapping Class Groups (MCG - A primer / Farb-Margalit)
6) Random Unitary Matrices & surfaces. (Magee-P)

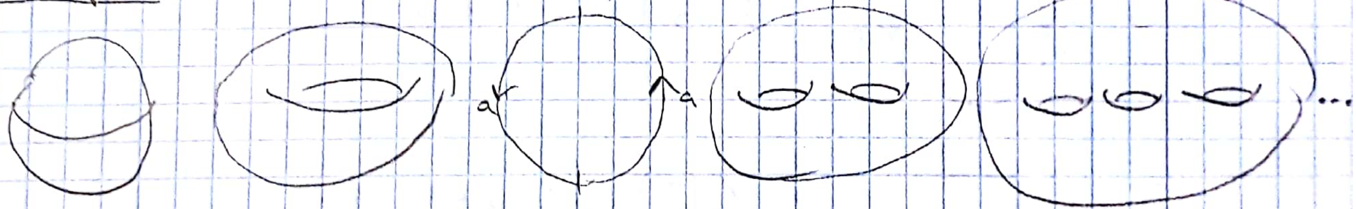
Lecture 1:

Classification of Surfaces:

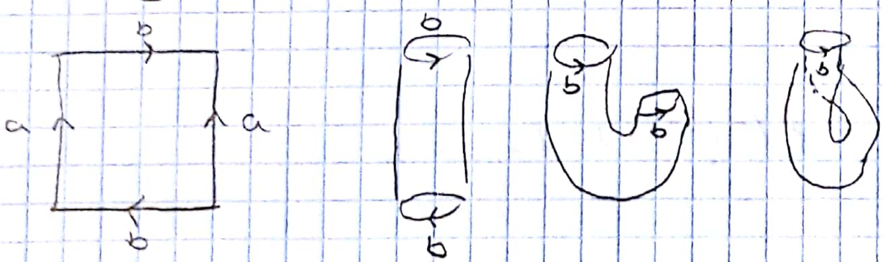
Def:

A surface is a 2-manifold. i.e a topological space which is:
1) locally Euclidean of dimension 2
2) Hausdorff. (counter example - plane with two origins)
3) second countable. Countable basis to the topology

Examples: (to closed Surfaces) & connected
compact, no boundary



the 2-sphere S^2 T $\mathbb{R}P^2$ $T\#T$ $T\#T\#T$



Klein bottle

$\mathbb{R}P^2$, KB cannot be embedded in $\mathbb{R}^3$ (Alexander, 1923)

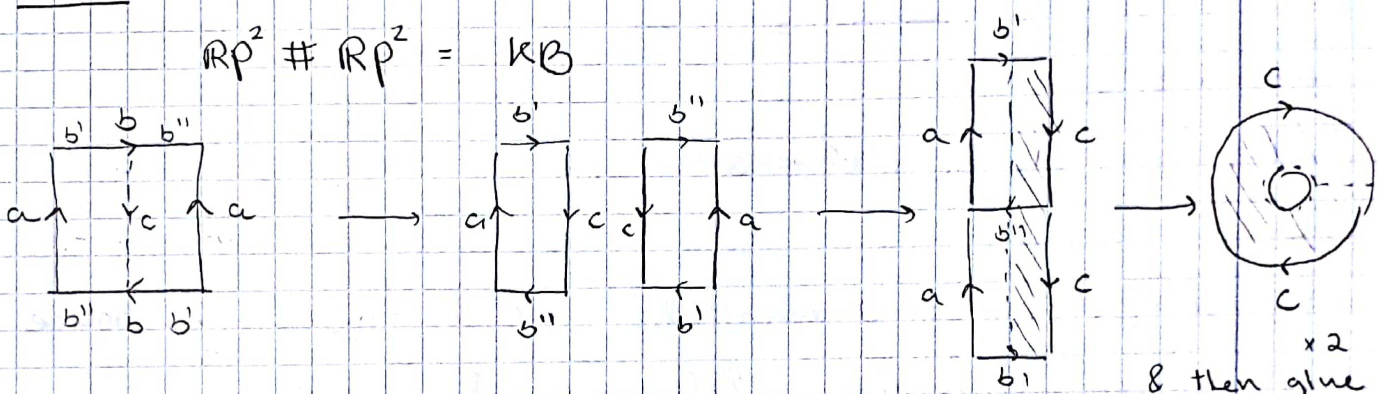
Connected Sum:

S_1, S_2 surfaces, the connected sum $S_1 \# S_2$ is obtained by removing a small piece from each & gluing along the boundary.

$$S^2 \# \underset{\substack{\uparrow \\ \text{any surface}}}{S} = S$$

Claim:

$$\mathbb{R}P^2 \# \mathbb{R}P^2 = KB$$



× 2

& then glue the holes in the middle.

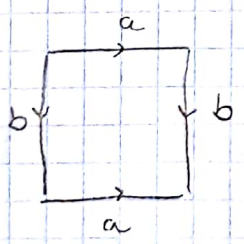
Euler Characteristics:

If a surface S is constructed as a simplicial / CW complex then $\chi(S) \overset{def}{=} \#\text{vertices} - \#\text{edges} + \#\text{faces}$.

S^2 :

$\chi(S^2) = 2 - 1 + 1 = 2$

T :



$\chi(T) = 1 - 2 + 1 = 0$

$\chi(\mathbb{R}P^2) = 1 - 1 + 1 = 1$

$$\chi(S_1 \# S_2) = \chi(S_1) + \chi(S_1) + \chi(S_2) - 2$$

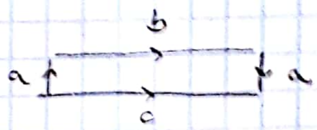
$$\begin{cases} \chi(S_1 \setminus \text{Disc}) = \chi(S_1) - 1 \\ \chi(S_2 \setminus \text{Disc}) = \chi(S_2) - 1 \end{cases}$$

$$\Rightarrow \chi(\underbrace{T\#\ldots\#T}_{\text{times } n}) = -2(n-1) = 2-2n$$

$$\chi(\underbrace{\mathbb{R}P^2\#\ldots\#\mathbb{R}P^2}_{n \text{ times}}) = n\cdot 1-(n-1)2 = 2-n$$

<u>Orientability</u>:

Möbius band:



<u>Def</u>:

A surface S is <u>not-orientable</u> if it contains a Möbius band (embedded).

$\Leftrightarrow \exists$ a closed curve in the surface